a, $a+x$, $a+2x$, $a+3x$, ... $a+(n-1)x$, is an arithmetical progression of n terms, their sum of squares is $a^2+(a+x)^2+(a+2x)^2+(a+3x)^2+$&c....
$+[a+(n-1)x]^2=S=na^2+(n^2-n)ax+\frac{1}{6}(2n^3-3n^2+n)x^2$ by method of difference. This sum is made=a rational square by the method given in Encyclopedia Britannica, Vol. I, Algebra article 121, 9th edition. When n is a square number $na^2+(n^2-n)ax+\frac{1}{6}(2n^3-3n^2+n)x^2=(n^{\frac{1}{2}}a+hx)^2$

$=na^2+2ahn^{\frac{1}{2}}x+\frac{1}{6}(2n^3-3n^2+n)x^2$. Then $x=\dfrac{2ahn^{\frac{1}{2}}-(n^2-n)a}{\frac{1}{6}(2n^3-3n^2+n)-h^2}=\frac{3}{2}$ for $a=1$,

$n=9$, $h=14$. The algebraic value of x in expressions for S_1 after clearing denominatives gives a general solution, the numerical $\frac{3}{2}$, gives
$2^2+5^2+8^2+11^2+14^2+17^2+20^2+23^2+26^2=48^2$.

Also solved by *F. P. Matz. G. B. M. Zerr, J. H. Drummond, C. D. Schmitt, J. Scheffer, and M. A. Gruber*

PROBLEMS.

30. Proposed by COOPER D. SCHMITT, Knoxville, Tennessee.

A and B are two integers, A consisting of $2m$ figures each being 1, and B consisting of m figures each being 4. Prove that $A+B+1$ is a square.

31. Proposed by M. A. GRUBER, War Department, Washington, D. C.

How many scalene triangles, of integral sides, can be formed with an altitude of 12? How many isosceles triangles?

AVERAGE AND PROBABILITY.

Conducted by B.F.FINKEL, Kidder, Missouri. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

17. Proposed by A. L. FOOTE, No. 80, Broad St. New York.

A person 30 years of age has an annuity for 10 years, the present worth of which is \$1000, provided he lives but ten years; for, if he dies, the annuity ceases What is the annuity worth, on the assumption that 75 out of every 4385 persons die annually, between the ages 30 and 40 years?

Solution by B. F. FINKEL, A. M., Professor of Mathematics and Physics, Kidder Institute, Kidder, Missouri.

Let S=the annuity, the present value, P, of which, for $10,=n$, years is \$100.

Then $P=\frac{S}{R-1}\left(\frac{R^n-1}{R^n}\right)$, whence $S=\frac{rPR^n}{R^n-1}$, where $R=1+r$. If we assume $r=.0$, we find $S=\$149.0294$ nearly.

The limit of B's life is $4385\div75=58\frac{7}{15}$ years, $=l$. $\therefore$ the probability that B will be living at the end of 1 year is $\frac{l-1}{l}$; at the end of two years, $\frac{l-2}{l}$; at the end of three years, $\frac{l-3}{l}$; etc.

The present worth of $S,=\$149.20207$, due in 1, 2, 3, 4, etc., years is $\frac{S}{R}$, $\frac{S}{R^2}$, $\frac{S}{R^3}$, etc., to $\frac{S}{R^n}$. The present worth of S due at the end of any year multiplied by the probability of B's living to the end of that year is the actual value of S.

$$\therefore\ S'=\frac{S(l-1)}{Rl}+\frac{S(l-2)}{lR^2}+\frac{S(l-3)}{lR^3}+\ldots\frac{S(l-n)}{lR^n}\ldots(1).$$

$$\frac{S'}{R}=\frac{S(l-1)}{lR^2}+\frac{S(l-2)}{lR^3}+\frac{S(l-3)}{lR^4}\ldots\frac{S(l-n)}{lR^{n+1}}\ldots(2),\text{ by multiplying (1) by }\frac{1}{R}.$$

$$\therefore\ S'\left(\frac{R-1}{R}\right)=\frac{S(l-1)}{Rl}-\frac{S}{lR^2}-\frac{S}{lR^3}-\ldots-\frac{S(l-n)}{lR^{n+1}},$$

$$=\frac{S}{l}\left\{\frac{l}{R}-\frac{1}{R}-\frac{1}{R^2}-\frac{1}{R^3}\ldots\frac{1}{R^n}-\frac{l-n}{R^{n+1}}\right\},$$

$$=\frac{S}{l}\left\{\frac{l}{R}-\frac{l-n}{R^{n+1}}-\frac{1}{R}-\frac{1}{R^2}\ldots\frac{1}{R^n}\right\},$$

$$=\frac{S}{l}\left\{\frac{l}{R}-\frac{l-n}{R^{n+1}}-\frac{1}{R}\left[1-\frac{1}{R^n}\right]\div\frac{R-1}{R}\right\},$$

$$=\frac{S}{R}\left\{\left(1-\frac{l-n}{lR^n}\right)-R\left(1-\frac{1}{R^n}\right)\div(R-1)l\right\},$$ whence $S'=S/r\{[1-(l-n)/lR^n]-R(1-1/R^n)/rl\},=\$911.881029$.

No solutions of this problem were received from our contributors.

18. Proposed by H. W. DRAUGHON, Clinton, Louisiana.

The probability that A will speak the truth is twice the probability that B will, in an independent statement, speak the truth; but, if A exerts his influence, the probability is that B will agree with him in any statement. What is the probability of the truth of their concurrent testimony, the chances being equal that A may or may not be interested in the matter?

Solution by P. H. PHILBRICK, C. E., Lake Charles, Louisiana.

1. Suppose that A is not interested in the matter. Let x=the probability of the truth of any one of B's statements. Then $2x$=the probability of the truth of any one of A's statements. The event did occur if both witnesses tell the truth the probability of which is $x\times2x=2x^2$.